# **Puritans' Home School Curriculum**

# INTRODUCTION TO LOGIC

# **TEACHER'S MANUAL**

J. Parnell McCarter

"The science of reasoning is of very great service in searching into and unraveling all sorts of questions that come up in Scripture... The validity of logical sequences is not a thing devised by men, but it is observed and noted by them that they may be able to learn and teach it; for it exists eternally in the reason of things, and has its origin with God."-Augustine

### **FIRST EDITION**

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The Puritans' Home School Curriculum

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# **SECTION ONE: COURSE INSTRUCTIONS**

#### Purpose

*A Puritans' Home School Curriculum Introduction to Logic* provides an introduction to the discipline of logic from a reformed Christian perspective. It incorporates use of the free on-line textbook and exercises of *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at <u>http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/</u>.

#### Prerequisites

Ideally a student will have completed at least one year of algebra before taking this course.

#### Check-Off List

Students should record when they have completed assignments on their check-off list, and teachers should record grades on the check-off lists. An assignment consists of readings and exercises. Teachers should grade the exercises for completeness and correct answers. Masters of the check-off lists have been included in this manual, from which copies can be made and distributed to students.

#### Assignments

An assignment consists of readings and exercises. Teachers should grade the exercises for completeness and correct answers. Each assignment covers at least one topic associated with the study of logic. Masters of the assignments have been included in this manual, from which copies can be made and distributed to students.

#### Grading

The average grade of all the assignments should be calculated in order to determine the overall course grade.

# **SECTION TWO: COURSE CHECK-OFF LIST**

# A Puritans' Home School Curriculum INTRODUCTION TO LOGIC

Student Name: \_\_\_\_\_

## Assignment Check-Off List

Assignment #	TOPIC	ASSIGNMENT COMPLETED? (X)	GRADE
1	What is Logic?		
2	The Language of Logic		
3	The History of the Discipline of Logic		
4	Statements and Logical Operators		
5	Logical Equivalence, Tautologies, and Contradictions		
6	The Conditional and the Biconditional		
7	Tautological Implications and Tautological Equivalences		
8	Rules of Inference		
9	Arguments and Proofs		
10	Predicate Calculus		
11	Logical Fallacies		
12	Understanding Logic		
13	Deductive and Inductive Reasoning		
	Assignment Grade Average		

**Note**: Grading in this course should be done on a 100-point scale, with letter grades assigned as follows:

Letter Grade	Score on 100-Point Scale
A+	97 - 100
А	94 - 96
A-	90 - 93
B+	87 - 89
В	84 - 86
B-	80 - 83
C+	77 - 79
С	74 - 76
C-	70 - 73
D	60 - 69
F	0 - 59

# **SECTION THREE: ASSIGNMENTS**

## **ASSIGNMENT 1 : WHAT IS LOGIC?**

#### **Reading:**

At the beginning of this introductory course on logic, it is appropriate that we first understand exactly what it is we are studying. So let's consider the term *logic* itself.

The term *logic* comes from the Greek word *logos*. Now *logos* is a term you should be familiar with, because it is found in such well known Bible passages as John 1:1 ("In the beginning was the Word [i.e., *Logos*], and the Word [i.e., *Logos*] was with God, and the Word [i.e., *Logos*] was God.") Of course, John 1:1 is referring to God the Son, who is Jesus Christ. Apparently, logic is such an inherent quality of God the Son, that the word of God uses the term as a name for God the Son, just as other names are elsewhere used for Him like 'Prince of Peace' and 'Counselor'. John chapter 1 informs us of some of those qualities of Jesus Christ which make it appropriate to call Him *Logos*. There we read how He is "the light that shineth in darkness." And we read how "grace and truth came by Jesus Christ." And we read how He "hath declared" God. Truth and the declaration of the truth are inherent qualities of God the Son, and He stands in opposition to error and falsehood. Right and truthful thinking, which is reasonable thinking, is an important characteristic of Christ, and a central feature of Christianity. Christianity is thus logical, and logic is right and truthful (i.e., reasonable) thinking.

The word of God scorns such nonsensical notions as "everything is relative". (Of course, the proposition itself is self-contradictory, for if the proposition were absolutely true, then not everything would truly be relative.) It also rejects such nonsense as "man cannot know truth". (Of course, the proposition itself is self-contradictory, for if man could know the proposition were true, then man could know a truth.) Pilate's question, "what is truth?", receives no scriptural sympathy. No, the Bible recognizes these are only foolish ploys by wicked men who want to hide from the truths of God's word and God Himself. "The fool hath said in his heart, there is no God." To deny God is ultimately to deny the reality of truth itself, which was indeed Pilate's implied excuse for not performing his duty. It is true, if there were no omniscient, omnipresent God who had revealed Himself to finite man, then truth would be beyond man's grasp. And since logic is conditioned upon the existence of truth (versus falsehood), to deny God is ultimately to sink into irrationality and the abandonment of logic. But Christianity is logical, while rebellious men are often illogical.

The book of Proverbs repeats over and over how we are to be wise. The Apostle Paul speaks of our *reasonable* service to God, in light of His mercies to us. We also read how the Apostle reasoned with his hearers. The sound exercise of reason is incorporated in the idea of logic. And God employs logic in His word to teach us to think wisely.

Fundamental to logic is the principle of non-contradiction. Contradictory propositions cannot both be true. For example, it is impossible that these two propositions can both be true:

Proposition 1 : Jesus was the Messiah.

Proposition 2 : Jesus was not the Messiah.

The word of God does not countenance relativism with regards to these propositions. In fact, scripture does not countenance positions contrary to any of the Biblical articles of faith, which is why the Apostle Paul insisted, "brethren, mark them which cause divisions and offenses contrary to the doctrine which ye have learned." So scripture implicitly teaches the principle of non-contradiction as a doctrine inherent in all of its doctrines and precepts. God thus commands men to be logical, as He is logical.

We should therefore understand logic as an attribute of God which is characterized by perfect reasonableness and thus free of contradiction and error. God's reasoning, and God's reasoning alone, sets the standards for logic. He is "the Way, *the Truth*, and the Life." As His intelligent creatures, it is our duty to seek to imitate His reasoning. So we are commanded: "be ye therefore perfect, even as your Father which is in heaven is perfect" (Matthew 5:48).

But, alas, ours is an age which takes extraordinary efforts to construct an edifice of knowledge absent any reference to God. (How else should we explain the effort to understand man's origins from naturalistic evolutionary processes operating on chance?) We should therefore not be surprised with the following sample of definitions of *logic* found in various contemporary dictionaries:

#### From *The American Heritage*® *Dictionary of the English Language*, Fourth Edition:

1. The study of the principles of reasoning, especially of the structure of propositions as distinguished from their content and of method and validity in deductive reasoning.

- 2. a. A system of reasoning: Aristotle's logic.
  - b. A mode of reasoning: By that logic, we should sell the company tomorrow.
  - c. The formal, guiding principles of a discipline, school, or science.
- 3. Valid reasoning: Your paper lacks the logic to prove your thesis.

4. The relationship between elements and between an element and the whole in a set of objects, individuals, principles, or events: There's a certain logic to the motion of rush-hour traffic.

- 5. Computer Science.
  - a. The nonarithmetic operations performed by a computer, such as sorting, comparing, and matching, that involve yes-no decisions.
  - b. Computer circuitry.
  - c. Graphic representation of computer circuitry.

[Middle English, from Old French logique, from Latin logica, from Greek logik (tekhn),

(art) of reasoning, logic, feminine of logikos, of reasoning, from logos, reason. See leg- in Indo-European Roots.]

From Webster's Revised Unabridged Dictionary, © 1996, 1998 :

\Log"ic\, n. [OE. logike, F. logique, L. logica, logice, Gr. logikh` (sc. te`chnh), fr. logiko`s belonging to speaking or reason, fr. lo`gos speech, reason, le`gein to say, speak. See Legend.]

1. The science or art of exact reasoning, or of pure and formal thought, or of the laws according to which the processes of pure thinking should be conducted; the science of the formation and application of general notions; the science of generalization, judgment, classification, reasoning, and systematic arrangement; correct reasoning.

From WordNet ® 1.6, © 1997 Princeton University :

the branch of philosophy that analyzes inference
reasoned and reasonable judgment; "it made a certain kind of logic"
the principles that guide reasoning within a given field or situation; "economic logic requires it"; "by the logic of war"
a system of reasoning [syn: logical system, system of logic]

Again, you will note that the definitions above are absent any theological reference. They acknowledge that logic, to use the definition from *Webster's Revised Unabridged Dictionary*, is "the science or art of exact reasoning, or of pure and formal thought, or of the laws according to which the processes of pure thinking should be conducted", but they fail to note that this is an inherent attribute of God, and that logic's existence is predicated upon God's existence. In this they greatly err.

#### **Exercise:**

Write a paragraph explaining in your own words the definition of *logic*. Compose this paragraph without the aid of the reading above in front of you when you write it.

### **ASSIGNMENT 2: THE LANGUAGE OF LOGIC**

All logic is expressed in symbols, or symbolic language. The symbolic language of logic comes in generally one of two forms. One form of symbolic language is ordinary spoken and written language, consisting of words. In this form of symbolic language, a word represents some thing, quality, or relation. For example, the word "dog" is a symbol that represents a 'highly variable domestic mammal closely related to the common wolf.' And the word 'red' is a symbol that represents a 'color whose hue resembles that of blood or of the ruby or is that of the long-wave extreme of the spectrum.' And the word 'is' is a symbol that represents the relation of equality. Obviously the logic contained in scripture is expressed primarily in this form of symbolic language. And the ancient Greek philosopher Aristotle's work on logic was also expressed in this way.

Here is an example of a logical argument expressed in ordinary language:

"George Washington was the first president of the United States. The first president of the United States was a resident of Mount Vernon. Therefore, George Washington was a resident of Mount Vernon."

Another form of symbolic language is mathematical language. Scripture alludes to this form of symbolic language, but it is not primarily written in this form of symbolic language. In an article in *Trinity Review* entitled "Math and the Bible" (see http://www.trinityfoundation.org/reviews/journal.asp?ID=027a.html ), J. C. Keister notes the many scriptural references to mathematics and mathematical language. Here is a sample of them, listed in his article:

"There are at least 150 references to arithmetic and geometry in the Old and New Testaments. To get an idea of some of these references, turn to Genesis where it says:

> When Adam had lived one hundred and thirty years, he became the father of a son in his own likeness, according to his image, and named him Seth. Then the days of Adam after he became the father of Seth were eight hundred years, and he had other sons and daughters. So all the days that Adam lived were nine hundred and thirty years, and he died (Genesis 5:3-5 NASB).

Among other things, this particular passage states that: 130 + 800 = 930.

An example of multiplication is contained in the New Testament, where it says:

And when they had come to Capernaum, those who collected the two drachma tax came to Peter, and said, "Does your teacher not pay the two drachma tax?" He said, "Yes." And when he came into the house, Jesus spoke to him first, saying, "What do you think, Simon? From whom do the kings of the earth collect customs or poll-tax, from their sons or from strangers?" And upon his saying, "From strangers," Jesus said to him, "Consequently the sons are exempt. But lest we give them offense, go to the sea, and throw in a hook, and take the first fish that comes up; and when you open its mouth, you will find a stater. Take that and give it to them for you and me" (Matthew 17:24-27 NASB).

Now, a stater is equivalent to four drachmas. Therefore, the passage is saying (among other things), that:

(2 drachmas/person) x (2 persons) = 4 drachmas, or more simply still,

$$2 \ge 2 = 4.$$

A subtraction problem is contained in:

In the fourth year the foundation of the house of the Lord was laid, in the month of Ziv. And in the eleventh year, in the month of Bul, which is the eighth month, the house was finished throughout all its parts and according to all its plans. So he was seven years in building it" (1 Kings 6:37-38 NASB).

Or, 
$$11 - 4 = 7$$
.

There is reference to the magnitude of pi (see 1 Kings 7:23-26) wherein the diameter and circumference of a circular bath are specified. It should be noted that the breadth of the container brim needs to be taken into account, 18 at which point it is clear that the value of pi obtained by dividing the circumference by the corrected diameter is within 1 percent of the actual value of pi. Since the measurements themselves are not absolutely precise (an error of 1/8 percent in the diameter measurement would account for the difference in the calculated value and actual value of pi), the correspondence is remarkable indeed.

Fractions are mentioned in Leviticus 27:27 and 32, and inequalities are either mentioned or implied in Matthew 12: 41-47 and Genesis 18:24-32. So it appears that the basic operations of arithmetic are presumed in various scriptural passages.

The Axioms of Arithmetic

We have seen evidence of the use of mathematics in Scripture. In addition, the rules of arithmetic are presumed. To see how this is so, let us examine the basic axioms of arithmetic:

1. $a + 0 = a$ (additive identity)
2. $a + b = b + a$ (commutative law of addition)
3. $(a + b) + c = a + (b + c)$ (associative law of addition)
4. a x $1 = a$ (multiplicative identity)
5. $ab = ba$ (commutative law of multiplication)
6. (ab)c = $a(bc)$ (associative law of multiplication)
7. $a(b + c) = ab + ac$ (distributive law of addition)
8. If $a = b$ , then $b = a$ (reflexive law)
9. If $b = c$ , then $b + a = c + a$ (identical addition operation)
10. If $b = c$ , then $ab = ac$ (identical multiplication operation)
11. $a + (-a) = a - a = 0$ (definition of -a)
12. a x $1/a = 1$ (a pi) (definition of $1/a$ )

The methods used to show that these axioms are illustrated in Scripture are basically the same as those used for any scriptural exegesis. Scripture is used to clarify Scripture, equivalent statements (mathematical in this case) are substituted where necessary, and any established generalization is used to help establish other generalizations (axioms in this case). Let us illustrate this commutative concept with the law of addition:

> For from now on five members in one household will be divided, three against two, and two against three (Luke12:52 NASB).

This passage is a clear illustration of the axiom that

a + b = b + a; specifically, it states that 3 + 2 = 2 + 3.

A second illustration of one of the axioms is the following:

Rule 3: Associative Law of Addition: (a + b) + c = a + (b + c)

(i.e., parentheses in addition processes don't matter):

The sons of Elioenai: Hodaviah, Eliashib, Pelaiah, Akkub, Johanan, Delaiah and Anani—seven in all (1 Chronicles 3:24 NIV).

Or, 1 + 1 + 1 + 1 + 1 + 1 + 1 = 7."

So while the symbolic language of logic we find in the Bible is primarily expressed in ordinary written language, there are clearly references in scripture to mathematical language. Both are useful tools of logic.

Mathematical language typically follows more formalized and uniform rules than ordinary language, and it is typically more efficient in its use of symbols than ordinary language. For example, in ordinary language various words can be used to express the relation of equality: "is", "are", "am", etc. But in mathematical language one symbol is uniformly employed to express this relation: "=".

Let's go back to our earlier example of a logical argument in ordinary language:

"George Washington was the first president of the United States. The first president of the United States was a resident of Mount Vernon. Therefore, George Washington was a resident of Mount Vernon."

We could express the same argument in mathematical language as follows:

```
a = bb = ca = c
```

Where a is "George Washington".

Where b is "the first president of the United States". And where c is "a resident of Mount Vernon".

#### **Exercise:**

Write your own logical argument in ordinary language which follows the pattern of the following argument in mathematical language:

a < b b < c -----a < c

# ASSIGNMENT 3: THE HISTORY OF THE DISCIPLINE OF LOGIC

#### **Reading:**

Contrary to what you would read in a humanistic textbook on logic, logic as a discipline did not begin with the Greek philosophers. Rather, the discipline of logic by man began as man pondered the word of God. When God spoke with Adam in Adam's beginning, man was thus confronted with pure logic. God's words of instruction and command were predicated upon logic, and consisted in pure logical propositions. Satan's words to man, and specifically to the woman, were illogical. They suggested God had lied, which is logically impossible, for God is true and the Truth. He cannot lie and be true to Himself. Eve's thinking became incoherent and illogical, and in this condition she sinned against God. The words of God to man, including the account of man's creation and fall, were compiled by Moses in writing. And as this divine revelation was studied, in such a manner was logic studied. Logic was thus integrated with theology, as it properly should be.

We should not pursue logic apart from God's revelation, lest we slip into error regarding the nature of logic, or lest we reach false conclusions based upon supposed logic. There are many ways in which man in history has fallen into irrationality by failing to base his system of reason firmly on the Bible. For instance, some have denied God's existence, some have denied human depravity, some have denied divine sovereignty, some have reduced that which exists to the material, some have adopted relativism, etc. Humanity, and especially fallen humanity, depends upon the revealed word of God in order to think reasonably.

The very notion that man can attain a reasonable system of knowledge apart from the divine revelation of scripture is itself unreasonable. (Humanism is the vain effort to construct such a system of knowledge apart from divine revelation.) Man is finite, and not omniscient. All human philosophies ultimately rest upon certain foundational propositions which cannot themselves be proved. Hence, all human world-views are ultimately pre-suppositional in nature. What distinguishes reformed Christianity from all other philosophies is that it and it alone contains the following three marks:

- 1. It acknowledges the requirement of pre-suppositional faith.
- 2. It is consistent with the history of human experience.
- 3. It is internally coherent and logically consistent.

Dr. W. Gary Crampton, in his review of Dr. Gordon Clark's textbook entitled *Logic* (see http://www.fpcr.org/blue\_banner\_articles/ReviewClarkLogic.htm), has summarized the way we should think about scripture as teaching logic thus:

"...logic is embedded in Scripture. The very first verse of the Bible, 'in the beginning God created the heavens and the earth,' necessitates the validity of the most fundamental law of logic: the law of contradiction (A is not non-A). Genesis 1:1 teaches that God is the Creator of all things. Too, it says that he created 'in the beginning.' It does not teach, therefore, that God is not the Creator of all things, nor does it maintain that God created all things 100 years after the beginning. The verse assumes that the words God, beginning, created, and so forth, all have definite meanings. It also assumes that they do not mean certain things. For speech to be intelligible, words must have univocal meanings. What makes the words meaningful, and revelation and communication possible, is that each word conforms to the law of contradiction.

This most fundamental law of logic cannot be proved. For any attempt to prove the law of contradiction would presuppose the truth of the law and therefore beg the question. Simply put, it is not possible to reason without using the law of contradiction. In this sense, the laws of logic are axiomatic. But they are only axiomatic because they are fixed or embedded in the Word of God.

Also fixed in Scripture are the two other principal laws of logic: the law of identity (A is A), and the law of the excluded middle (A is either B or non-B). The former is taught in Exodus 3:14, in the name of God itself: "I AM WHO I AM." And the latter is found, for example, in the words of Christ: "He who is not with me is against me" (Luke 11:23).

Logic, then, is embedded in Scripture. This is why Scripture, rather than the laws of logic, is selected as the axiomatic starting point of Christian epistemology. Similarly, God is not made the axiom, because all of our knowledge of God comes from Scripture. "God," as an axiom, without Scripture, is merely a name. Scripture as the axiom defines God.

As we are taught in the Bible, man is the image of God (Genesis 1:26,27). God "formed man of the dust of the earth and breathed into his nostrils the breath of life; and man became a living soul" (Genesis 2:7). Adam became a type of soul that is superior to that of non-rational animals (2 Peter 2:12). Man, as God's image bearer, is a rational being (Colossians 3:10). This is why the apostle Paul could spend time "reasoning" with his auditors "from the Scriptures" (Acts 17:2).

Moreover, because Christ is the Logos who "gives [epistemological] light to every man who comes into the world" (John 1:9), we are to understand that there is a point at which man's logic meets God's logic. In fact, John 1:9 denies that logic is arbitrary; it also denies polylogism, i.e., that there may be many kinds of logic. According to John, there is only one kind of logic: God's logic. And the Logos gives to every image bearer of God the ability to think logically.

Man, then, has the capacity to think logically and to communicate with God. God created Adam with a mind structured in a manner similar to his own. In the Scripture, God has given

man an intelligible message, "words of truth and reason" (Acts 26:25). God has also given man language that enables him to rationally converse with his Creator (Exodus 4:11). Such thought and conversation would not be possible without the laws of logic. Logic is indispensable to all (God-given) human thought and speech. This being so, we must insist that there is no "mere human logic" as contrasted with a divine logic. Such fallacious thinking does disservice to the Logos of God himself."

So God's words to man, and man's thoughts upon His words and His creation, were not confined to the people of God. These were also considered and pondered by the pagan descendants of Adam and later of Noah. Of course, the account of God's words to man transmitted outside of scriptural revelation became corrupted, owing to man's sin and ignorance. So pagan man labored in his study of logic at a great disadvantage. Nevertheless, as the image-bearers of God- albeit as corrupted image-bearers - all men, pagan and elect, could study logic. And it was in fact the pagan Greeks that sought to systematize logic.

The Greeks then are undoubtedly the most famous scholars of logic in ancient history. To the extent their systemization of logic was consistent with scripture, then it has been useful for humanity. The Greek philosopher Aristotle wrote in his Organon a systematic treatise on logic. His work in particular had a heavy influence on philosophy, science and religion through the Middle Ages. Much of this influence was detrimental to the cause of truth (its pre-suppositional basis failing to acknowledge the necessity of God's word as the foundation for human knowledge and reason), but its significant influence in Western philosophy is nevertheless a reality. Roman Catholic philosophy, in contrast to reformed Christian philosophy, grants the ability of fallen man to reason rightly independent of divine revelation. So Roman Catholic philosophers have granted greater authority to Aristotelian philosophy than reformed Christian philosophers.

The philosopher Leibniz in the 17th century advocated the use of mathematical language for the study of logic. And in 1847 with G. Boole's book *The Mathematical Analysis of Logic* and A. DeMorgan's book *Formal Logic*, much of logic as it is generally studied today became part of mathematics. This also stretched the definition of mathematics not only to be about numbers (arithmetic) and shapes (geometry), but also to encompass any subject that can be expressed symbolically with precise rules of manipulation of those symbols. Such mathematical language is a useful tool for learning logic, but it cannot obviate ordinary language as well in this study. Already in this introductory course in logic I have communicated ideas about it to you through use of ordinary language. Man thinks and communicates by means of ordinary language as well as mathematical language. This is the case in general, and it is the case in the study of logic.

#### **Exercise:**

Write a paragraph explaining in your own words the history of the discipline of logic. Compose this paragraph without the aid of the reading above in front of you when you write it.

# **ASSIGNMENT 4: STATEMENTS AND LOGICAL OPERATORS**

#### **Reading:**

For the next seven assignments of this course, we shall be reading sections from the on-line textbook *Introduction to Logic* by Stefan Waner and Steven R. Costenoble, and then performing the exercises provided in their textbook for each section.

Section 1 from *Introduction to Logic* by Stefan Waner and Steven R. Costenoble concerns statements and logical operators. Statements are also called propositions. A statement is any declarative sentence which is either true (T) or false (F). We refer to T or F as the truth value of the statement.

In assignment 2 we had considered these 3 propositions:

- 1. George Washington was the first president of the United States.
- 2. The first president of the United States was a resident of Mount Vernon.
- 3. George Washington was a resident of Mount Vernon.

Each of these declarative sentences could be assigned a true value of either true (T) or false (F), so all three sentences are statements. It so happens that all three of these statements have in actuality a truth value of true (T).

Statements can be represented by letters. Thus we could say:

p = George Washington was the first president of the United States.

q = The first president of the United States was a resident of Mount Vernon.

r = George Washington was a resident of Mount Vernon.

We can form new statements from old ones in several different ways. For example, starting with the statement " George Washington was the first president of the United States," we can form the negation of the statement. The negation of the statement is: "George Washington was not the first president of the United States." We denote the

negation of p by ~p, read "not p." What we mean by this is that, if a is true, then ~a is false, and vice-versa. Since a ("George Washington was the first president of the United States") is true, for example, then ~p ("George Washington was not the first president of the United States") is false. The symbol ~ is an example of a logical operator.

Now read section 1 in *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at website http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logic1.html .

#### **Exercises:**

Do the on-line exercises for section 1 of *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at:

http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logicex1.html .

# ASSIGNMENT 5: LOGICAL EQUIVALENCE, TAUTOLOGIES, AND CONTRADICTIONS

#### **Reading:**

Two statements can be logically equivalent. We say that two statements are logically equivalent if, for all possible truth values of the variables involved, both statements are true or both are false.

An example of two logically equivalent statements is a statement and its doubly negated statement. A double negation is represented as follows:

~(~p)

So a statement p is logically equivalent to  $\sim(\sim p)$ .

Going back to this statement about George Washington ("George Washington was the first president of the United States"), we can see that it is logically equivalent to its double negation ("It is not the case that George Washington was not the first president of the United States"). The two negatives as it were cancel one another out. So a statement is logically equivalent to its double negation.

A tautology is a statement which is true by its logical form alone. It is true in all cases because of its logical form.

An example of a tautology would be this statement:

"A black cat is a cat that is black."

Or stated another way:

"If the cat is black, then it is a black cat."

Or expressing the statement more symbolically, this is a tautology:

if a is true, then a is true.

It does not matter what statement a is, in all cases the statement is true.

Another tautology, symbolically expressed, would be the following:

#### if $\sim(\sim p)$ , then p.

In all cases the statement is true, based upon its logical form.

A contradiction is a statement that asserts or implies both the truth and falsity of something. For example, here is a contradictory statement:

"George Washington was the first president of the United States, and George Washington was not the first president of the United States."

Contradictory statements are meaningless and illogical.

As we have already noted, an inherent attribute of God is that He is logical, and He created a logical universe. An important aspect of being logical is being non-contradictory. So when Jesus Christ said, "I am the Way, the Truth, and the Life", He implied that He was not False. He warned about the coming of false christs in the future, but He affirmed that He was the true christ. And it would be wrong to say Jesus Christ is not Truth, because this would contradict that He is the Truth. Truth would lose all meaning if truth and falsehood were both truth. It would be just as impossible for God to be contradictory as it would be for Him to lose His attributes of omniscience or omnipresence.

Reformed Biblical Christianity is the only world-view which is logically consistent and coherent, without internal contradictions. All other world-views (whether Islam, atheism, Hinduism, etc.) distinguish themselves as false because they contain internal contradictions, which is contrary to the character of God. They sink into incoherent mysticism and irrationality due to their logical inconsistency. Modern Western civilization itself is sinking into such irrationality due to its embrace of secular humanism. Secular humanism cannot, for instance, logically account for absolute standards of universal right and wrong- absolute standards which every human society needs to function properly.

Even among Christian denominations, we can judge which is right based upon its logical consistence (i.e., non-contradiction) with scripture. For instance, Roman Catholicism shows itself to be flawed with its doctrine of the re-sacrifice of Christ in its Romish Mass, which contradicts Hebrews 9:25-28 ("nor yet that He should offer Himself often...So Christ was *once* offered to bear the sins of many...") and Hebrews 10:10 ("By the which will we are sanctified through the offering of the body of Jesus Christ *once for all.*") Although Roman Catholicism officially adheres to the infallibility of scripture, many of its own doctrines contradict scripture.

Historically some Christians have objected to this insistence upon logical consistency. They assert that it imperils Christian orthodoxy, including such doctrines as the trinity of God and divine sovereignty yet human responsibility. But these examples are really no proofs at all,

because there is nothing contradictory in either the doctrine of the Trinity or the doctrine of divine sovereignty and human responsibility or any other doctrines taught in scripture. The God who has revealed Himself in scripture is the "God of order", and He is "not the author of confusion". He is the God who insists: "let your yea be yea; and your nay, nay". As Trinitarian Christians, when we assert that there is one God, we are not asserting that there is one *unitarian* God. That would indeed contradict the doctrine of the Trinity. No, we are asserting that there is one God that is trinitarian in nature. And when we assert that God the Son (Jesus Christ) took upon Himself man's nature, we are *not* asserting that Jesus' human nature was indistinct from His divine nature, or that He lost His divine nature. That would indeed be contradictory, for human nature is finite but divine nature is infinite (in knowledge, presence, power, etc.). Although these natures were united in the one person of Jesus Christ, they (necessarily) remained distinct in Him, for the attributes of divinity *are* distinct from the attributes of humanity. While it is not contradictory to assert that divine nature is or became human nature.

Similarly, divine sovereignty and human responsibility are not contradictory ideas. As Jonathan Edwards so excellently proved in his treatise on the freedom of the will, those who assert contradiction do so because of wrong or fuzzy conceptions of human (free) will. That man has a will to desire certain things in no wise contradicts that God sovereignly decrees all things that come to pass, including the will of man. And man's will constantly and intelligently desiring right or wrong things implies human responsibility for such choices.

Such matters as the doctrine of the Trinity and the doctrine of divine sovereignty and human responsibility are unquestionably hard to understand. And, undoubtedly, man will never understand them as fully as God comprehends them. But we must distinguish issues which are difficult to comprehend from statements that are contradictory. After all, when we assert our belief in the doctrine of the Trinity we are *not* asserting that there are three Gods and yet there is only one God. Now that would be contradictory!

Now read section 2 in *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at website http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logic2.html .

#### **Exercises:**

Do the on-line exercises for section 2 of *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at:

http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logicex2.html .

# ASSIGNMENT 6: THE CONDITIONAL AND THE BICONDITIONAL

#### **Reading:**

A conditional statement is one which reads either as "if p, then q" or as "p implies q". In such a statement p is called the antecedent or hypothesis, and q is called the consequent or conclusion.

An example of a conditional statement is as follows:

"If George Washington was the first president of the United States, then the first president of the United States was a man."

In such conditional statements, whenever the antecedent is true, then the consequent must be true.

A statement is said to be biconditional if both "p implies q" and "q implies p". Or another way of putting the biconditional statement is "if p, then q" and "if q, then p".

Now read section 3 in *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at website http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logic3.html.

#### **Exercises:**

Do the on-line exercises for section 3 of *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at:

http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logicex3.html .

# ASSIGNMENT 7: TAUTOLOGICAL IMPLICATIONS AND TAUTOLOGICAL EQUIVALENCES

#### **Reading:**

In a previous assignment we had considered tautologies. They are, as you will recall, statements which are always true due to their logical form. In this assignment we will consider tautological implications.

Tautological implications are tautologies of the form "if A, then B".

One form of tautological implication involves direct reasoning, and another form of tautological implication involves indirect reasoning.

Modus Ponens (or direct reasoning) presents itself in this form:

"If p implies q, and if p is true, then q must be true."

For example:

"If that man lives at Mount Vernon implies he is George Washington, and if that man indeed lives at Mount Vernon, then that man is truly George Washington."

Modus Tollens (or indirect reasoning) presents itself in this form:

"If p implies q, and q is false, then so is p."

For example:

"If that man lives at Mount Vernon implies he is George Washington, and if that man is not George Washington, then that man does not live at Mount Vernon."

Tautological equivalences are tautologies of the form "A is logically equivalent to B". A and B are (possibly compound) statements that are logically equivalent.

Some examples of tautological equivalences are:

p is logically equivalent to  $\sim(\sim p)$ 

(p and q) is logically equivalent to (q and p)

Now read section 4 in *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at website http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logic4.html .

#### **Exercises:**

Do the on-line exercises for section 4 of *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at:

http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logicex4.html .

## **ASSIGNMENT 8: RULES OF INFERENCE**

#### **Reading:**

A proof is a way of showing how a conclusion follows from a collection of premises. For example, suppose someone asked you to prove the following argument:

Premise 1: If p implies q. Premise 2: p is true.

Conclusion: q is true.

The proper response would be "the rule of Modus Ponens".

That is because the rule of Modus Ponens says that q is logically equivalent to:

(p implies q) and p

The rule of Modus Ponens is frequently displayed in scripture. Consider, for example, the account of the Fall. God had warned man:

"If you eat the forbidden fruit, you shall surely die."

Man indeed ate the forbidden fruit on a certain day. And just as we would expect, the curse of death was pronounced upon him that day. God had held man responsible, because man was expected to reason as follows:

Premise 1: If man eats the forbidden fruit, man shall surely die.

Premise 2: Man ate the forbidden fruit.

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Conclusion: Man shall surely die.

Similarly, God warned the Israelites: "if thou wilt not hearken unto the voice of the Lord thy God, to observe to do all his commandments and his statutes which I command thee this day; that all these curses shall come upon thee, and overtake thee" (Deuteronomy 28:15). The Israelites ended up not obeying the commandments of God. Therefore, the Lord cursed Israel, as He said He would: "…behold, I will bring evil upon this place, and upon the inhabitants thereof, even all the curses that are written in the book…Because they have forsaken me…therefore my wrath shall be poured out upon this place" (II Chronicles 34:24-25). And so God destroyed Israel as He said He would, by the hands of the Assyrians and the Babylonians.

The Modus Ponens argument proclaimed by God's prophets like Isaiah and Jeremiah runs thus:

Premise 1:If Israel rebels against God, then God will destroy Israel.Premise 2:Israel rebelled against God.

\_\_\_\_\_

Conclusion: God will destroy Israel.

In accordance with the divinely ordained rule of Modus Ponens, the prophets thus declared Israel's destiny.

Now, of course, there are other rules we have already learned besides just the rule of Modus Ponens. These are called rules of inference. A rule of inference is just an instruction for obtaining additional true statements from a list of true statements. And we find these other rules manifested in scripture as well.

Let's consider, for instance, the rule of Modus Tollens. This method of indirect reasoning is structured as follows:

p implies q ~q ~p

Jesus Christ employed the rule of Modus Tollens to refute the false assertions of His Pharisaic detractors. They claimed Jesus was casting out demons because He was possessed by Satan. He evinced the absurdity of their claim in Matthew 12:25-26 with the following implied argument:

Premise 1: If Satan (and those possessed by Satan) cast out Satan's demons, then Satan is divided against himself.

Premise 2: Satan is obviously not divided against himself (for nothing divided against itself can long stand).

\_\_\_\_\_

Conclusion: Satan (and those possessed by Satan) do not cast out Satan's demons.

In this same discourse Jesus also asserted the rule of non-contradiction, which is the same as not being divided against oneself. Clearly Christ would have regarded it as even more outrageous to assert that God is divided against Himself. God is logically consistent, and He does not do those things which do not accrue to His own glory.

The other rules of inference could be similarly illustrated in scripture.

Now read section 5 in *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at website http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logic5.html .

#### **Exercises:**

Do the on-line exercises for section 5 of *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at:

http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logicex5.html .

## **ASSIGNMENT 9: ARGUMENTS AND PROOFS**

#### **Reading:**

An argument is a list of statements called premises followed by a statement called the conclusion. An argument is valid if the conjunction of its premises implies its conclusion. In other words, validity means that if all the premises are true, then so is the conclusion. Validity of an argument does not guarantee the truth of its premises, so does not guarantee the truth of its conclusion. It only guarantees that the conclusion will be true if the premises are.

A proof is a way of convincing you that the conclusion follows from the premises, or that the conclusion must be true if the premises are. Formally stated, a proof of an argument is a list of statements, each of which is obtained from the preceding statements using one of the rules of inference. The last statement in the proof must be the conclusion of the argument.

Now read section 6 in *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at website http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logic6.html .

#### **Exercises:**

Do the on-line exercises for section 6 of *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at:

http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logicex6.html .

# **ASSIGNMENT 10: PREDICATE CALCULUS**

#### **Reading:**

A syllogism is a deductive scheme of a formal argument consisting of a major and a minor premise and a conclusion. Here is a sample syllogism:

Major Premise:Every sin is a transgression of the law.Minor Premise:Adultery is a sin.Conclusion:Adultery is a transgression of the law.

A useful tool for analyzing syllogisms is predicate calculus. Predicate calculus differs from the propositional calculus we have been studying in previous assignments. Predicate calculus allows us to mathematically analyze many arguments like the syllogism above using what are called universal quantifiers and existential quantifiers.

This is a very difficult chapter, but even if you cannot master it at this point in your education, it is good to become acquainted with it.

Now read section 7 in *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at website http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logic7.html.

#### **Exercises:**

Do the on-line exercises for section 1 of *Introduction to Logic* by Stefan Waner and Steven R. Costenoble at:

http://people.hofstra.edu/faculty/Stefan\_Waner/RealWorld/logic/logicex7.html .

# **ASSIGNMENT 11: LOGICAL FALLACIES**

#### **Reading:**

The Greek philosopher Plato is quoted as saying: "Arguments, like men, are often pretenders." A fallacy is simply a faulty argument—a pretender, so to speak. In the process of reasoning, there are two types of fallacies that occur: formal and informal. Formal fallacies deal with the actual form of the argument. When an argument is structured incorrectly it is fallacious. But even when an argument is formally correct it may still be informally fallacious. The conclusion may not actually follow from the premises due to a faulty gathering of information or some other mistake. Informal fallacies are the more common of the two types of fallacies.

The website www.summit.org/resources/Critical\_Thinking/logicandlogicalfallacies.htm offers a list of common informal fallacies, excerpted below:

#### Fallacies of Ambiguity

Communication can be difficult in our day. Difficulties arise from differing cultures, age groups, races, prejudices, and especially from differing worldviews. One of the most important ground rules that must be followed for clear communication is clear definitions. We may be unnecessarily frustrated if others misunderstand what we say because they either don't know what a word means, or we simply have not supplied clear definitions of our words. Several fallacies arise from unclear language. We'll look at three.

#### 1. Equivocation

The fallacy of equivocation occurs when we use different definitions for the same word, or when a word is taken in a different way than intended (a different definition). Many words have different meanings depending on their context. Consider: "I saw." Now, that could mean visual comprehension, or it could be a claim to profession (i.e. "I work in a saw mill. Therefore, I saw.") How about the word "pen?" Is it a writing utensil or an enclosure for animals? Consider the following examples:

"All men are created equal? If that were so, then there wouldn't be so many rich people."

"If all men are created equal, then why am I so short?"

The difficulty that arises in these examples is that the statement "all men are created equal" means that they are to be equally valued, as human beings. It was never intended to mean [in that context] that we are all clones of one another, or that we would have equal outcomes (a tenant of socialism)...

It should be noted that much of our humor rests in equivocations. In a humorous context, we call it a "play on words." Also, sometimes an equivocation can be intentional and witty, such as when Ben Franklin declared, "We must all hang together, or they will hang us separately." The word "hang" is intended to be understood quite differently in the two instances...

Special Instance. When Christians are witnessing to people who are bound up in the various pseudo-Christian religions (i.e. cults) of today, they need to be very careful to define their words so as not to be misunderstood. For example, while Mormons and Jehovah's Witnesses both use the name of Jesus Christ, they have completely different meanings. The Jehovah's Witness believes that Jesus was the first created being and was, in fact, Michael the Archangel before he became the man Jesus. The Mormons, on the other hand, believe that Jesus is literally our older brother from a pre-existence. Jesus is believed to be the firstborn of the Father and one of his many wives! Given these differences, we need to make sure that we dig deeper into the meanings of what people say and not stay at a superficial level of communication.6

#### 2. Amphibole

The fallacy of amphibole (pronounced with a long 'e') occurs when the ambiguity of words or phrases arises from their grammatical structure. One of the more famous examples of amphibole occurred on a wartime conservation poster. It read:

"Save Soap and Waste Paper"

Well? Are we to save both soap and paper, or only soap?

Here's another example:

"I live on the top floor; drop by some time."

What? Am I to visit, or climb to the top of the building and jump off?

As can be seen with these examples, words can be understood differently, depending upon their relationships to other words. (A good way to remember the name of this fallacy is to think of an amphibian—an animal that can live in two different surroundings, land and water. Words can sometimes do double duty, depending on their grammatical context.)

#### 3. Accent

The fallacy of accent occurs when the meaning of a sentence is changed through differing emphases, accents, or tones of voice. For example, a mere inflection of the voice can change the meaning of a sentence from a straightforward statement to a question: "I love you." to "I love you?" Even with the latter example one must wonder if the question is about love (I love you?), or a person (I love you?).

Consider this following example. It is said, "We should not speak ill of the dead." This can be understood in quite different ways depending on which words are accented:

We should not speak ill of the dead. But someone else can?

We should not speak ill of the dead. But we will anyway?

We should not speak ill of the dead. But we can think ill of them?

We should not speak ill of the dead. But we can speak ill of the living?

This sort of misunderstanding arises more often when we are reading than when we are listening to a speaker. The reason for this is that a good speaker will be able to use inflection to make his or her meaning clear. When we read, though, we tend to put emphasis where it seems right to us. We need to be careful in doing this.

#### Fallacies of Relevance

This section will deal with those fallacies that occur when something irrelevant to the question of truth is added to an argument in the attempt to persuade. These fallacies simply appeal to what is irrelevant to the question at hand.

#### 1. Appeal to Pity

Emotions are wonderful gifts from God. We can be happy and sad, joyful and brokenhearted, angry and forgiving. But human emotions were never intended to be guardians of the truth. The heart is to follow the head, not vice versa. The difficulty with this particular fallacy is that everyone is swayed by emotions from time to time. There is nothing necessarily wrong with this. We should be emotionally driven to follow the Lord, for example. We should also be emotionally driven to follow the truth, as well. But we should not let our emotions run our minds. In fact, we are told by the Apostle Paul to be transformed by the renewing of our minds (Romans 12:1-2). The fallacy of appeal to pity occurs when we determine right and wrong, true and false, by means of our emotions. While we may not come to wrong conclusions, more often than not, we do.

The issue of abortion is very emotional. Every side of the debate flourishes with emotionalism. One side tries to persuade the other in the name of "choice", another in the name of "life." But it is one thing to use slogans and clichés, it is quite another to provide clear solid reasons why your position is true...

#### 2. Genetic Fallacy

The genetic fallacy is committed when a person argues that something (or someone) is false (or bad) because of where it (he or she) came from. A very common example of this fallacy can be seen in the evolutionist objection to creation: "You're getting science from the Bible." The fallacy can be clearly seen in that evidence for creation should be examined, no matter where it comes from, before simply writing it off because a person believes in the Bible. What is the evidence?

Nathanael's initial response to Philip's proclamation, "Jesus of Nazareth" is a good example of this fallacy. Nathanael responded, "Nazareth! Can anything good come from there?" (John 1:46). Whether Jesus came from Nazareth was beside the point.

#### 3. Ad Hominem (argument to the person)

Another fallacy is called Ad Hominem—meaning argument to the person. This fallacy is committed when instead of dealing with what a person is arguing, one argues, for example, that the person is lacking in character. The reason this is fallacious is that a person's character has no bearing on the truth or falsehood of his or her claims. (A legitimate use of this type of argument can be found in a court of law. For example, witnesses must retain upstanding character for their testimony to be believed. If a witness is known to be an habitual liar, then there is good cause to doubt the integrity of his or her testimony. But reason to doubt his testimony does not prove that what he is saying is false! That must be checked out with the facts. Apart from valid instances such as this, there are many that areinvalid.)

In Matthew chapter 11 Jesus rebuts an ad hominem argument that was leveled against him. The Pharisees and Sadducees had long been trying to discredit Jesus and his message. One of the tactics they tried was to discolor his character. In verses 18 and 19 we read Jesus' rebuttal:

For John came neither eating nor drinking, and they say, `He has a demon.' The Son of Man came eating and drinking, and they say, `Here is a glutton and a drunkard, a friend of tax

collectors and "sinners." But wisdom is proved right by her actions.

Jesus not only points to the stubborn nature of that generation (they would not be pleased with either John or Jesus), but he makes the point that while they may attempt to demean his character they will be proved wrong and Jesus will be proved right. Not only are their objections falsely portraying Jesus and John, but Jesus makes the point that his life will be vindicated (proved right) by his actions.

Another type of ad hominem argument is the appeal to force. When someone says that you better agree with them, or else they will beat you up—that is an appeal to force! Forcing someone to agree with you does not make you right. Might does not make right!

## 4. Poisoning the Well

This fallacy (a sub-fallacy of ad hominem) occurs when a person discredits an opposing perspective without even considering any evidence. Name-calling is a good example of this fallacy. If you can discredit a person with a degrading name, even before they present their case, then you have "poisoned the well." No one need consider what they have to say. Of course, the question is not how you label them, but whether or not what they claim is true.

# 5. Appeal to Ignorance

This fallacy can occur in two ways. 1) To argue that something is true because it hasn't been proven to be false; or 2) to argue that something is false because it hasn't been proven to be true. Just because there is no proof against your position does not prove your position true. Likewise, just because a position has not been proven does not mean that it is false. Sometimes we just need to suspend final judgment until more evidence is in.

## 6. Fallacy of the Beard

This fallacy of the beard is committed when a person argues that you cannot come to a conclusion because one thing differs from another only in degree. The name of the fallacy derives from the difficulty of determining when exactly someone has a beard. Is it when one has a `five-o'clock shadow'? When the whiskers are one quarter of an inch long? Longer? Just when is one's facial hair long enough to be called a beard? Just because one cannot determine how long the hair has to be does not mean that three inches of facial hair cannot be called a beard, and a slight stubble should not. Just because the line is hard to draw does not mean that differences mean nothing...

# Fallacies of Presumption

Fallacies of presumption are those fallacies where someone holds to an unjustified conclusion. This is usually caused by overlooking, denying, evading, or distorting the facts.

## 1. Hasty Generalization

When you wish to make an argument for a certain position, you need to gather information and evidence as your support. In doing this, you must be very careful to gather enough evidence to actually support your conclusion. The fallacy of hasty generalization is committed when a person gathers too little information to support the conclusion being argued. Just because one or two taxi drivers are rude and obnoxious does not mean that you can generalize that all taxi drivers are just the same, or even that most are...

## 2. Sweeping Generalization

The fallacy of sweeping generalization is committed when one takes a general rule and applies it absolutely to all instances, not recognizing that there are exceptions. The generalization might be a very fair one, but the application in particular, uncommon, or unique instances may not be...

## 3. Faulty Dilemma

This fallacy is committed when a person argues that there are only a certain number of options, and you must choose between them, when in fact there are more options available. This fallacy is also called the "either/or fallacy," because it looks like you have to choose either this, or that. In John 9:2-3 the disciples posed a faulty dilemma when they asked concerning a man who had been blind from birth, "Rabbi, who sinned, this man or his parents, that he was born blind?" This is an either/or type of question. Instead of answering the question along one of the lines offered, Jesus denies both . . . and supplies a third. Jesus said, "Neither this man nor his parents sinned, but this happened so that the work of God might be displayed in his life."

## 4. Loaded or Complex Question

One very common attempt by unbelievers to stump believers is to ask the age old question:

"Can God create a rock so big that he can't lift it? Yes or No?"

How would you answer? If you answer yes, then God's omnipotence (all-powerfulness) is denied due to the fact that he can't lift the rock. But if you answer no, then God's omnipotence is denied because he can't create such a rock. But neither of these answers is satisfying to a Bible-believing Christian. So where does the problem lie? How is one to get out of this dilemma?

This example can be classified as the fallacy of a loaded question, or a complex

question. What if I asked you, "Have you stopped beating your wife yet?" Well, yes or no? If you answer yes, that implies that you have been beating her. And if you answer no, then you are still beating her! The problem lies in the question. Not all questions are good, fair questions. And this question is one of those that is simply not fair to ask (hopefully). You would have to respond that you have never beat your wife, and that the question presupposes that you have. You can't simply answer with a yes or a no. The question is wrong.

Now back to God and the big rock. You cannot answer this question with a simple yes or no. Either answer would make you deny what the Bible teaches about the nature of God. What you have to do is to show that the question is wrong; it is not a fair question at all...You see, by definition, since God is omnipotent (and that is what the Bible teaches), he could create the largest rock possible. Also, because God is omnipotent, he could lift the largest possible rock. The problem with the dilemma is the dilemma—it is faulty; the question was loaded. You cannot set the creative expression of an omnipotent being against the abilities of an omnipotent being. That would be just as illogical as asking whether or not God could create a square-circle. Of course, no such thing could be created. Therefore, it is not within the realm of reality to speak of such illusions. But such illusions do not in any way illustrate any limitation in God's power and abilities.

#### 5. False Cause

In Latin, this fallacy is called post hoc, ergo propter hoc, which literally means "after this, therefore because of this." This fallacy is committed when a person believes that just because one thing followed another there must be a causal connection...

## 6. Straw Man

The straw man fallacy occurs when a person misrepresents another's view so as to easily discredit it. This can happen intentionally or unintentionally. The image that this fallacy conjures up is that of a person building a straw man just to knock it over. Well, straw men seem to be easier to knock over than real men. One might say, "You say that the New Testament teaches that we are not under law, and that we are saved by grace through faith alone. Therefore, what you teach is that we can sin all we want after we are saved." This is clearly a straw man according to Paul in Romans 6:15ff.

## 7. Bandwagon Fallacy

This fallacy is committed when we appeal to a group of people to prove that something true or false, or right or wrong. Many times Americans fall into this trap. For example, some people think that certain sexual practices are justified because over 50% of the American public believes that it is all right. But we cannot determine right and wrong by majority vote. In India, a practice called satee was very common and adhered to by the majority of the people. This

practice entailed burning a widow —alive—along with the body of her deceased husband. Both of these examples show that we do not determine right and wrong by majority vote...

## 8. Two Wrongs Make a Right

"Well, Johnny did it too!" You've no doubt heard similar words before: someone trying to justify his actions based on the fact that it was done to him. But merely showing another's guilt does not prove your innocence. Just because someone else is wrong also doesn't make you right; it just makes both of you wrong. Two wrongs don't make a right, they just make two wrongs.

#### 9. Appeal to Authority

There are times when all of us need to appeal to authorities. Maybe you are not a pro when it comes to automobiles, so you appeal to a trained mechanic—an authority. There are times when we visit a doctor's office in order to get an authoritative opinion concerning our health. There are many valid appeals to authority. But there are also many invalid appeals to authorities. An appeal to authority can be fallacious when we appeal to someone who may indeed be an authority, but not in the field in question...

#### 10. Chronological Snobbery

This fallacy occurs when one appeals either to what is old, or to what is new, in the attempt to establish the truth (as if age were an indicator of truth!). Someone may appeal to what is traditional. "We have always done it this way, it must be right." In the end, though, there may be a better way. More often today, we hear an appeal to the "modern." "We moderns don't believe in the existence of God. That was for ages past when people believed in mythology." Merely because something is old or new does not make it right or true.

#### 11. Argument to the Future

This fallacy is committed when one appeals to the future to prove one's case. While there may indeed be evidence forthcoming, this is no ground for arguing that the case is true because there might be something in the future to vindicate that belief. You may be wrong.

"There is a way that seems right to a man, but the end thereof is death."

The Mormons (members of the Church of Jesus Christ of Latter-day Saints) sometimes argue that the Book of Mormon is just as historical as the Bible. The problem with this claim is that there is no direct evidence to support its unique elements (such as Christ visiting the Americas). (If there were direct evidence to support the historical accuracy of the Book of Mormon, such as archaeological sites of cities unique to the Book of Mormon, then we would expect maps published in the back of the book—like there are in most Bibles. But the fact is, the Mormon church has never published an official map of Book of Mormon lands or cities. Why not?) A Mormon may try to argue that the evidence will be found in the future, but this is no proof. How would they know of such future finds? This would be mere assertion.

Evolutionists are always caught in a bind when it comes to the fossil record. The reason for this is that transitional forms do not exist in the record. Most often the evolutionist will argue that, while the fossil record is incomplete (which assumes the case to be proved, and thus begs the question), someday the needed transitional forms will be found. But such assertions are fallacious because the transitional forms are needed to prove evolution, and merely appealing to the future is no proof that such proof will ever be found.

Another website ( http://phuakl.tripod.com/pssm/REASON.htm ) helpfully categorizes common informal and formal logical fallacies in alphabetical order by their Latin name:

#### ARGUMENTUM AD NUMERAM

A fallacy that asserts that the more people who support or believe a proposition then the more likely that that proposition is correct; it equates mass support with correctness - the fallacy that led to Man believing that the earth is flat for centuries.

#### ACCENTUS

One of the Fallacies of Ambiguity, which arises from the emphasis (the accent) placed on a word or phrase.

#### AFFIRMING THE CONSEQUENT

An argument based on a hypothetical statement, and the truth of the consequent to the truth of the antecedent. In the SYLLOGISM below, A is the antecedent and C is the consequent:

A implies C C is true <-- Affirming the consequent

Therefore: A is true

#### AMBIGUITY

An argument in the course of which at least one term (such as "rights") is used in different senses. Also known as equivocation. There are several types of "fallacies of ambiguity," including REIFICATION, EQUIVOCATION, AMPHIBOLY, COMPOSITION, DIVISION, and ACCENTUS.

#### AMPHIBOLY

A type of Fallacy of Ambiguity where the ambiguity involved is of an "amphibolous" (equivocal, uncertain) nature. Amphiboly is a syntactic error. The fallacy is caused by faulty sentence structure, and can result in a meaning not intended by the author. "The department store now has pants for men with 32 waists." (How many waists do you have?)

#### ARGUMENTUM AD ANTIQUITAM

A fallacy of asserting that something is right or good simply because it is old; that is, because "that's the way it's always been."

#### ARGUMENTUM AD BACULUM

An argument that resorts to the threat of force to cause the acceptance of the conclusion. Ad baculum arguments also include threats of fear to cause acceptance (e.g., "Do this or you'll go to Hell when you die!" or "I made him an offer he couldn't refuse.").

#### ARGUMENTUM AD CRUMENAM

Fallacy of believing that money is a criterion of correctness; that those with more money are more likely to be right. "If he's so stupid why is he so rich?" The reverse of a. ad crumenam is a. ad lazarum.

#### ARGUMENTUM AD HOMINEM

An argument that attempts to disprove the truth of what is asserted by attacking the speaker rather than the speaker's argument. Another way of putting it: Fallacy where you attack someone's character instead of dealing with issues. The two basic types of ad hominem arguments: are 1) abusive, and 2) circumstantial. This is the most common form of Logical Fallacy.

#### ARGUMENTUM AD IGNORANTIAM

An argument that a proposition is true because it has not been shown to be false, or vice versa. Ad ignorantium arguments are also known as "appeals to ignorance." This fallacy has two forms:

- 1. The statement is true, because it has not been proven false.
- 2. The statement is false, because it has not been proven true.

#### ARGUMENTUM AD LAZARUM

A fallacy of assuming that because someone is poor he or she is sounder or more virtuous than one who is wealthier. -- "responsible breeders don't make money." This fallacy is the opposite of the informal fallacy a. ad crumenam.

#### ARGUMENTUM AD MISERICORDIAM

An argument that appeals to pity for the sake of getting a conclusion accepted (or for fundraising).

#### ARGUMENTUM AD NAUSEUM

The incorrect belief that an assertion is more likely to be true the more often it is heard. An a. ad nauseum is one that employs constant repetition in asserting a a statement is the truth. Dr. Goebbel's Big Lie Theory.

#### ARGUMENTUM AD NOVITAM

A fallacy of asserting that something is more correct simply because it is new or newer than something else. Or that something is better because it is newer. -- "we've tried the other way for a while and it's failed, let's try something (anything) ." This type of fallacy is the opposite of a. ad antiquitam.

#### ARGUMENTUM AD NUMERAM

A fallacy that asserts that the more people who support or believe a proposition then the more likely that that proposition is correct; it equates mass support with correctness.

#### ARGUMENTUM AD POPULUM

An argument that appeals to the beliefs of the multitude. Another way of putting it: Speaker deals with passions of audience rather than with salient issues. This fallacy is also known as "Appeal to Tradition" Ad populum arguments often occur in 1) propaganda, 2) demagogy, and 3) advertising.

#### ARGUMENTUM AD VERECUNDIAM

An argument in which an authority is appealed to on matters outside his/her field of authority. (like veterinarians dispensing medical advice). a.ad verecundiam also refers to a fallacy of simply resorting to appeals to authority (like "Doctor" Tom Regan)

## BEGGING THE QUESTION (CIRCULAR REASONING)

An argument that assumes as part of its premises that the conclusion is true. Another way of saying this is: Fallacy of assuming at the onset of an argument the very point you are trying to prove. This Fallacy is also known by the Latin "PETITIO PRINCIPII" or "CIRCULUS IN PROBANDO."

# BIFURCATION

Also referred to as the "black and white" fallacy, bifurcation is the presentation of a situation or condition with ONLY TWO alternatives, whereas in fact other alternatives exist or can exist.

## COMPOSITION

An argument which assumes that a whole has a specific property solely because its various parts have that property. -- "Because ALF is a terrorist organization (and ALF is part of PETA) > all PETA members condone terrorism." Composition is a type of Fallacy of Ambiguity.

## CONVERTING A CONDITIONAL

Description: If A then B, therefore, if B then A. <<<NOT!!!

# CUM HOC ERGO PROPTER HOC

A fallacy of correlation that links events because they occur simultaneously; one asserts that because two events occur together they are causally related, and leaves no room for other factors that may be the cause(s) of the events. This fallacy is similar to the "post hoc" fallacy.

## DENIAL OF THE ANTECEDENT

An argument in which one infers the falsity of the consequent from the truth of a hypothetical proposition, and the falsity of its antecedent.

A implies B Not-A

Therefore: Not-B

## DIVISION

An argument in which one assumes that various parts have a property solely because the whole has that same property. Division is a type of Fallacy of Ambiguity -- the inverse of COMPOSITION.

## EQUIVOCATION

An argument in which an equivocal expression is used in one sense in one premise and in a different sense in another premise, or in the conclusion. Equivocal means 1) of uncertain significance; not determined, and 2) having different meanings equally possible. Equivocation is a type of Fallacy of Ambiguity. The opposite of equivocation is "UNOVOCATION," in which a word always carries the same meaning through a given context.

## FALLACY OF INTERROGATION

The question asked has a presupposition which the answerer may wish to deny, but which he/she would be accepting if he/she gave anything that would count as an answer. Any answer to the question "Why does this event happen?" presupposes that the event does indeed happen.

## FALSE ANALOGY

An analogy is a partial similarity between the like features of two things or events on which a comparison can be made. A false analogy involves comparing two things that are NOT similar. Note that the two things may be similar in superficial ways, but not with respect to what is being argued.

## IGNORATIO ELENCHI

An argument that is supposed to prove one proposition but succeeds only in proving a different one. IGNORATIO ELENCHI stands for "pure and simple irrelevance."

## ILLICIT PROCESS

A syllogistic argument in which a term is distributed in the conclusion, but not in the premises. One of the rules for a valid categorical syllogism is that if either term is distributed in the conclusion, then IT MUST BE DISTRIBUTED IN THE PREMISES. There are two types of Illicit Process: Illicit Process of the Major Term and Illicit Process of the Minor Term.

## PLURIUM INTERROGATIONUM - COMPLEX QUESTIONS

A demand for a simple answer to a complex question.

#### NON CAUSA PRO CAUSA

An argument to reject a proposition because of the falsity of some other proposition that SEEMS to be a consequence of the first, but really is not.

#### NON-SEQUITUR - DOES NOT FOLLOW

An argument in which the conclusion is not a necessary consequence of the premises. A conclusion drawn from premises that provide no logical connection to it.

#### PETITIO PRINCIPII

The same as "Begging the Question" This argument assumes its conclusion is true but DOES NOT SHOW it to be true. Petitio principii has two forms:

P is true, because P is true.
P is true, because A is true. And A is true because B is true.
And B is true because P is true.

#### POST HOC, ERGO PROPTER HOC

An argument from a premise of the form "A preceded B" to a conclusion of the form "A caused B." Simply because one event precedes another event in time does not mean that the first event is the cause of the second event. This argument resembles a fallacy known as a HASTY GENERALIZATION.

#### QUATERNIO TERMINORUM

An argument of the syllogistic form in which there occur four or more terms. In a standard categorical syllogism there are ONLY THREE TERMS: a subject, a predicate, and a middle term.

#### **RED HERRING**

A fallacy when irrelevant material is introduced to the issue being discussed, such that everyone's attention is diverted away from the points being made, and toward a different conclusion. It is not logically valid to divert a chain of reasoning with extraneous points.

#### REIFICATION

To reify something is to convert an abstract concept into a concrete thing. Reification is a Fallacy of Ambiguity. Reification is also sometimes known as a fallacy of "HYPOSTATIZATION".

#### SECUNDUM QUID (HASTY GENERALIZATION)

An argument in which a proposition is used as a premise without attention given to some obvious condition that would affect the proposition's application. This fallacy is also known as the "HASTY GENERALIZATION." It is a fallacy that takes evidence from several, possibly unrepresentative, cases to a general rule; generalizing from few to many. NOTE THE RELATION TO STATISTICS: Much of statistics concerns whether or not a sample is representative of a larger population. The larger the sample size, the better the representativeness. Note also that the opposite of a hasty generalization is a sweeping generalization.

#### SHIFTING THE BURDEN OF PROOF

The burden of proof is always on the person making the assertion or proposition. Shifting the burden of proof, a special case of "ARGUMENTUM AD IGNORANTIUM," is a fallacy of putting the burden of proof on the person who denies or questions the assertion being made. The source of the fallacy is the assumption that something is true unless proven otherwise.

#### SPECIAL PLEADING

Special pleading is a logical fallacy wherein a double standard is employed by the person making the assertion. Special pleading typically happens when one insists upon less strict treatment for the argument he/she is making than he or she would make when evaluating someone else's arguments.

#### STRAW MAN

It is a fallacy to misrepresent someone else's position for the purposes of more easily attacking it, then to knock down that misrepresented position, and then to conclude that the original position has been demolished. It is a fallacy because it fails to deal with the actual arguments that one has made.

#### SWEEPING GENERALIZATION

Also known by the Latin term "DICTO SIMPLICITER", a Sweeping Generalization occurs when a general rule is applied to a particular situation in which the features of that particular situation render the rule inapplicable. A sweeping generalization is the opposite of a hasty generalization.

## TWO WRONGS MAKE A RIGHT (TU QUOQUE)

Two wrongs never add up to a right; you cannot right a wrong by applying yet

another wrong. Such a fallacy is a misplaced appeal to consistency. It is a fallacy because it makes no attempt to deal with the subject under discussion.

## UNDISTRIBUTED MIDDLE

A syllogistic argument in which the middle term of a categorical syllogism is not distributed in AT LEAST ONE of the premises.

We should learn to recognize these common forms of fallacy and avoid them in our own thinking.

# **Exercises:**

Specify the fallacy committed in each of the following statements:

- 1. My plants died after I watered them, so my watering them must have caused them to die.
- 2. The people on that airplane must have been very wicked, because all of them died when the plane crashed.
- 3. I know I will go to heaven because the priest said I would.
- 4. Since it is all right for the government to put someone to death for murder, it must be all right for me personally to kill someone if I am persuaded they committed murder.
- 5. Children like to ride bikes. Sally likes to ride a bike. Sally must be a child.
- 6. John hit me, so I have a right to hit him back.
- 7. All Christians are hypocrites. Just look at how these television evangelists are hypocrites.
- 8. Sue is quite hot with anger. Hot objects burn you if you touch them. Sue therefore will burn you if you touch her.
- 9. I am right and you are wrong, because everyone in the classroom agrees with me.
- 10. You should plant your tomatoes in the shade like my dentist recommends.

# ASSIGNMENT 12: UNDERSTANDING LOGIC

## **Reading:**

In the first assignment in this course, we considered the definition of logic, and we looked at the definitions found in various dictionaries. One dictionary that was excluded was *The Free On-line Dictionary of Computing*. Its definition of logic was excluded because at that stage you would have been unprepared to understand it. But now, having gone through the previous assignments, you should be able to understand it.

From The Free On-line Dictionary of Computing, © 1993-2003 Denis Howe :

"A branch of philosophy and mathematics that deals with the formal principles, methods and criteria of validity of inference, reasoning and knowledge.

Logic is concerned with what is true and how we can know whether something is true. This involves the formalization of logical arguments and proofs in terms of symbols representing propositions and logical connectives. The meanings of these logical connectives are expressed by a set of rules which are assumed to be self-evident.

Boolean algebra deals with the basic operations of truth values: AND, OR, NOT and combinations thereof. Predicate logic extends this with existential and universal quantifiers and symbols standing for predicates which may depend on variables. The rules of natural deduction describe how we may proceed from valid premises to valid conclusions, where the premises and conclusions are expressions in predicate logic.

Symbolic logic uses a meta-language concerned with truth, which may or may not have a corresponding expression in the world of objects called existence. In symbolic logic, arguments and proofs are made in terms of symbols representing propositions and logical connectives. The meanings of these begin with a set of rules or primitives which are assumed to be self-evident. Fortunately, even from vague primitives, functions can be defined with precise meaning.

Boolean logic deals with the basic operations of truth values: AND, OR, NOT and combinations thereof. Predicate logic extends this with existential quantifiers and universal quantifiers which introduce bound variables ranging over finite sets; the predicate itself takes on only the values true and false. Deduction describes how we may proceed from valid premises to valid conclusions, where these are expressions in predicate logic.

Carnap used the phrase "rational reconstruction" to describe the logical analysis of thought. Thus logic is less concerned with how thought does proceed, which is considered the realm of psychology, and more with how it should proceed to discover truth. It is the touchstone of the results of thinking, but neither its regulator nor a motive for its practice."

It should be noted that the essentially humanistic definition above calls "primitives" what we called "pre-suppositions" or "axioms" in earlier assignments. But it incoherently waffles as to whether these should be conceived as *absolute* rules describing "how it [i.e., thought] *should* proceed to discover truth", or merely human conventions "assumed to be self-evident". By not rooting them in the inherent character of the omniscient, logical, infinite God, humanistic philosophy fails to explain why we conceive logic in an *absolute* sense. If it resorts to citing logic's success in human experience, it still cannot explain their use as an absolute standard, for man's experience is merely finite. The only being that would know in an absolute sense how thought should proceed to discover truth, is an omniscient God who knows truth and who knows how to attain it. In the absence of a logical God who sets the standard for thought, the use of the term "should" with respect to human thought is incoherent. It is similar to the inconsistent way in which materialistic evolutionists speak in terms of "good" and "bad" while at the same time asserting that the world is nothing more than chance reactions of atoms.

## **Exercises:**

- 1. Define Boolean logic.
- 2. Describe why humanistic analyses of logic fail (e.g., Carnap's rational reconstruction of human logic).

# **ASSIGNMENT 13: DEDUCTIVE AND INDUCTIVE REASONING**

#### **Reading:**

We have considered in this introductory course in logic the logic of God, and the reflection of His logic in the logic of His word. Biblical Christianity authenticates logic. Having thus been authenticated, we are warranted to employ logic in our theological studies. For this reason we read in the first chapter of the Westminster Confession: "the whole counsel of God, concerning all things necessary for his own glory, man's salvation, faith, and life, is either expressly set down in scripture, or by good and necessary consequence may be deduced from scripture..."

Deductive reasoning is inferential reasoning in which the conclusion about particulars follows necessarily from general or universal premises. Deductive reasoning is mandated in God's word. For example, the prophets of God rebuked Judah and Israel for their sins, arguing from the general principles found in the Ten Commandments, and applying them to specific instances in the life of Judah and Israel. In His Sermon on the Mount, Jesus Christ also showed the applicability of the moral law to various matters of the heart, from general principles revealed in the Old Testament. Without an omniscient God who has revealed truth to man, deductive reasoning would be vain because only an omniscient God can know those principles which are absolutely and universally true.

Some Christians have erred in insisting that our theology can and should go no further than the express statements of scripture. They insist upon an *explicit* proof text in order to believe a doctrine, and they reject arguments based upon deduction from the statements of scripture. But this methodology of scriptural study ignores the scriptural imperative of deduction.

Another form of reasoning is inductive reasoning. Inductive reasoning is arriving at a generalized conclusion from particular instances. Inductive conclusions are generally valid because God maintains and upholds the physical laws of the universe. But the only being in the universe who can engage in inductive reasoning with absolute certitude of conclusion is the omniscient, omnipresent God. So we must be careful to recognize the limits of inductive reasoning for humans. There are various times in history when God has supernaturally intervened, working a miracle which breaks the normal pattern of physical laws. Thus, Jesus' disciples were amazed when He could feed multitudes with a few fish and a few loaves of bread, and end up with more food than He started. Uniformitarianism errs in assuming uniformity of physical laws on a universal basis. It is a humanistic pre-supposition that is contrary to the word of God and to the history of the world revealed in the word of God.

God's word teaches us how we ought to reason wisely and not foolishly. And the fear of the Lord is the beginning of wisdom.

# **Exercises:**

- 1. Define deductive reasoning.
- 2. Define inductive reasoning.